

A Note on Singular Instantons

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Abstract

We point out the existence of some singular, radial, spin-0 instantons for curvature-quadratic gravity theories. They are complex.

We consider Euclidean gravity theories described by the Lagrangian density

$$\sqrt{g}(\alpha R_{ik}R^{ik} + \beta R^2). \quad (1)$$

For these theories there are solutions of the Euler-Lagrange equations of the form

$$g_{\mu\nu}(x) = \delta_{\mu\nu} (r^2)^\varepsilon \quad (2)$$

for $\varepsilon = -2, -1 \pm i/\sqrt{3}$. Here

$$r^2 = \sum_{i=1}^4 x_i^2. \quad (3)$$

For $\varepsilon = -2$ the instanton is also a (well-known, trivial) instanton of the Einstein action.

For the two complex values of ε this is not true, and the values of the metric are complex.

For $\beta = -\frac{1}{3} \alpha$, any $g_{\mu\nu}$ of the form

$$g_{\mu\nu}(x) = \delta_{\mu\nu} f(x) \quad (4)$$

satisfies the Euler-Lagrange equations; one is dealing with a conformal gravity theory.